B. Sc. Part II (Hons.) 4th Paper

Diff. Eqns (contd)

(LDE CC)

Q. Solve

$$(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$$

Soln The given equation

$$(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$$

for CF

$$D^3 - 5D^2 + 7D - 3 = 0$$

$$\Rightarrow D^3 - D^2 - 4D^2 + 4D + 3D - 3 = 0$$

$$\Rightarrow D^2(D-1) - 4D(D-1) + 3(D-1) = 0$$

$$\Rightarrow (D-1)(D^2 - 4D + 3) = 0$$

$$\Rightarrow (D-1)(D-1)(D-3) = 0$$

$$\Rightarrow D = 1, 1, 3$$

$$\therefore CF = (C_1 + C_2 x)e^x + C_3 e^{3x}$$

for PI

$$PI = \frac{1}{D^3 - 5D^2 + 7D - 3} e^{2x} \cosh x$$

$$\because \cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\Rightarrow PI = \frac{1}{2(D^3 - 5D^2 + 7D - 3)} e^{2x} (e^x + e^{-x})$$

$$\Rightarrow PI = \frac{1}{2(D-1)^2(D-3)}\left(e^{3x}+e^{x}\right)$$

$$\Rightarrow PI = \frac{1}{2(D-1)^2(D-3)}e^{3x} + \frac{1}{2(D-1)^2(D-3)}e^{x}$$

$$= \left[\frac{1}{2(3-1)^2}\right]\frac{1}{D-3}e^{3x} + \left[\frac{1}{2(1-3)}\right]\frac{1}{(D-1)^2}e^{x}$$

$$= \frac{1}{8}\cdot\frac{1}{(D-3)}e^{3x} - \frac{1}{4(D-1)^2}e^{x}$$

$$= \frac{e^{3x}}{8}\cdot\frac{1}{[(D+3)-3]} - \frac{e^{x}}{4}\cdot\frac{1}{[(D+1)-1]^2}$$

$$= e^{3x}\frac{1}{8D}\cdot x^0 - \frac{e^{x}}{4D^2}x^0$$

$$= e^{3x}\cdot\frac{1}{8}\int dx - \frac{e^{2x}}{4}\int\left[\int dx\right]dx$$

$$= \frac{xe^{3x}}{8} - \frac{e^{2x}}{4}\cdot\frac{x^2}{2} = \frac{xe^{3x}}{8} - \frac{x^2e^{2x}}{8}$$

Hence, complete solution is given by

$$y = CF + PI$$

$$\Rightarrow y = (c_1 + c_2x)e^{x} + c_3e^{3x} + \frac{1}{8}\left(xe^{3x} - x^2e^{2x}\right)$$